Wave-Based Turing Machine: Time Reversal and Information Erasing

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(Received 17 January 2016; revised manuscript received 20 June 2016; published 26 August 2016)

The investigation of dynamical systems has revealed a deep-rooted difference between waves and objects regarding temporal reversibility and particlelike objects. In nondissipative chaos, the dynamic of waves always remains time reversible, unlike that of particles. Here, we explore the dynamics of a wave-particle entity. It consists in a drop bouncing on a vibrated liquid bath, self-propelled and piloted by the surface waves it generates. This walker, in which there is an information exchange between the particle and the wave, can be analyzed in terms of a Turing machine with waves as the information repository. The experiments reveal that in this system, the drop can read information backwards while erasing it. The drop can thus backtrack on its previous trajectory. A transient temporal reversibility, restricted to the drop motion, is obtained in spite of the system being both dissipative and chaotic.

DOI: 10.1103/PhysRevLett.117.094502

In physics, time reversal symmetry, i.e., the invariance of the dynamical equations under the transformation \( t \rightarrow -t \), is a general property of conservative systems. But, there is a widely studied fundamental difference [1] in the temporal reversibility of waves and particles. For waves, temporal reversibility is preserved even in the presence of chaos. This yields surprising possibilities as observed in optical phase conjugation [2], time-reversed acoustics [3,4], microwaves [5], elastic waves [6], or surface waves [7,8]. For particles, as demonstrated in, e.g., billiards, the temporal reversibility is destroyed in the chaotic regimes by the sensitivity to initial conditions.

Is time reversibility restored for a particle dynamics when it is piloted by a wave? In the present work, we investigate this question experimentally using a walker, a dynamical entity associating a drop bouncing on a vertically oscillating bath with the waves it generates [9–14]. We first show that an imposed phase shift between the bouncing motion and the surrounding waves leads naturally to a reversal of the instantaneous drop velocity. Surprisingly, in chaotic regimes, the same phase shift leads not only to a reversal of the instantaneous velocity, but also to a reversal of the motion along complex trajectories. This is equivalent to a temporal reversibility, unexpected in a system both dissipative and chaotic. We show that this property emerges from a dynamical erasing process of the pregenerated wave field. Eventually, we revisit the dynamics of these walkers in terms of writing, storing, reading, and erasing processes. We show that this system implements the basic elements of a Turing machine using standing waves as a global information repository.

Walkers are obtained in the experimental setups usually devoted to the study of the Faraday instability [15]. A bath of silicon oil of viscosity \( \nu = 2 \times 10^{-2} \) Pas is oscillated vertically with an acceleration \( \gamma(t) = \gamma_m \cos(2\pi f_0 t) \) and a frequency \( f_0 = 80 \) Hz. When the imposed oscillating acceleration exceeds a threshold \( \gamma_F \approx 4.5g \), waves appear spontaneously at the bath surface. These are parametrically forced Faraday standing waves oscillating at half the forcing frequency: \( f_f = f_0/2 \) [16]. Below but close to the Faraday instability threshold \( (4g < \gamma_m < \gamma_F) \), a drop of typical diameter \( D = 600 \) \( \mu m \) of the same oil is observed to bounce at half the forcing frequency \( f_f = f_0/2 \). Correlatively, the drop excites damped Faraday waves [11]. In this regime, the drop becomes propelled by its interaction with the waves it emits. At each collision with the bath, the drop receives a kick in a direction determined by the local slope of the interface. The resulting wave-induced force can be written \( \mathbf{F}_m = -C \nabla h \), where \( C \) is a coupling constant and \( \nabla h \) is the surface height gradient evaluated at the impact point [10]. It leads to a propelled motion at a velocity \( V_0 \approx 10 \) mm/s.

As sketched in Fig. 1, the drop vertical motion being subharmonic, it can have two different phases relative to the forcing oscillation. Having chosen arbitrarily an origin of time, we can distinguish odd and even periods of the forcing [respectively labeled \( A \) and \( B \) in Fig. 1(a)]. In normal conditions, the bouncing is strictly subharmonic so that a given drop hits the surface during either the \( A \) or the \( B \) periods. The emitted waves are also subharmonic and temporally synchronized with the bouncing phase of the drop. An abrupt change of the drop bouncing from, e.g., phase \( A \) to phase \( B \), therefore generates a \( \pi \) shift between the drop periodic motion and the preexisting wave. The \( \pi \) shift is induced by a brief and controlled disturbance of the forcing oscillation, as sketched in Fig. 1(a). The amplitude of the sinusoidal input \( \gamma(t) \) is increased from \( \gamma_m \) to \( \gamma_m + \Delta \gamma_m \) during two periods \( T_0 \) of the bath oscillation.

Regardless of the bouncing phase, the drop receives one
The possibility of imposing abruptly a $\pi$ shift to a walker motion is an intriguing problem with potential implications for several fields of research, including physical sciences, mathematics, and even economics. This technique could be used to study the effects of abrupt changes on dynamic systems, leading to new insights into the behavior of these systems. Further research could focus on applying this method to more complex systems, such as those involving multiple degrees of freedom or systems with nonlinear dynamics.
forces the drop to return on a predetermined trajectory so that during a finite time, the motion is reversed, even though the dynamics is dissipative. It is the first observation of an effect theoretically predicted by Devaney [19] that a dynamical system (not necessarily conservative) is reversible if there exists a transformation $G$ in phase space that reverses the direction of time and is also an involution (i.e., $G$ composed with itself yields identity). In the present study, a $\pi$ shift is such an involution of the current dynamics. In the very high memory limit ($Me \approx 150$), the motion of a walker is affected by fluctuations of its velocity modulus. It very appear to be a hindrance to a further increase of the reversibility time.

An insight into this time reversal phenomenon is provided by investigating the wave field. It can be recorded as seen from above at 800 fps with a fast camera phase locked on the bath oscillations. The effect of the $\pi$ shift on the wave field is best observed by extracting from these recordings two films at half the forcing frequency and corresponding to two observations strobed in phases $A$ and $B$, respectively (see Supplemental Material 1 [20]). Figures 3(a) and 3(b) show two instantaneous wave fields observed when the drop is moving (a) forward and (b) backward, respectively. During its initial motion, the bouncing of the drop builds up a wave field. It can be observed by comparing Figs. 3(a) and 3(b) that the whole wave field associated with the return motion seems to be of smaller amplitude than that of the forward motion. This effect can be analyzed quantitatively by computing the wave field from the drop trajectory [11] (see detail in Supplemental Material 2 [20]). The surface height at time $t$ in position $\tilde{\rho}$ is given by

$$h(\tilde{\rho}, t) = h_0 \sum_{t_n > t_e} J_0(k_F||\tilde{\rho} - \tilde{r}(t_n)||) e^{-(t-t_n)/(MeT_F)} - h_0 \sum_{-\infty < t_n < t_e} J_0(k_F||\tilde{\rho} - \tilde{r}(t_n)||) e^{-(t-t_n)/(MeT_F)},$$

(1)

the forward and the backward motions is then measured from the distance $d$ between the droplet positions at two times symmetrical with respect to the $\pi$ shift $d(\Delta t)$, as sketched in Fig. 2(f). A histogram of the distances $d(\Delta t)$ is plotted in Fig. 2(e) as a function of the elapsed time $\Delta t$ from the $\pi$ shift. The distances $d$ remain small during a time $\Delta t$ of the order of $Me/2$ for all the imposed $\pi$ shift. Control experiments have then been done using the same experimental parameters but without performing any $\pi$ shift. The observed motion is a highly chaotic trajectory. This chaos had been investigated using a Poincaré return map [18]. In this regime, two trajectories with neighboring initial conditions diverge from each other. In contrast, a $\pi$ shift

![FIG. 2. Trajectories observed before (solid blue line) and after a $\pi$ shift (solid red line) for various types of orbital motions. (a) Stable circular orbit observed for $Me = 100$ and $\Lambda = \Lambda_{2,2} = 0.9$. (b) Stable lemniscate observed at $Me = 100$ and $\Lambda = \Lambda_{2,0} = 0.75$. (c),(d) Chaotic regimes for $Me = 70$, $\Lambda = 0.49$ and $Me = 180$, $\Lambda = 0.82$. In the two latter cases, the drop motion is reversed on a trajectory length of the order of $Me$. (e) Histogram of the temporal evolution of the distance $d(t)$ [as defined in (f)] of the drop position at two times symmetrical with respect to the time $t_e$ of the phase shift. This color-coded histogram was obtained from $N = 250$ trajectories. Time is expressed in number of bounces.](Image 1)

![FIG. 3. Direct visualisation of the wave field observed during the (a) forward and the (b) backward motion of the drop. The forward (blue arrow) and backward (red arrow) trajectories have been superimposed.](Image 2)
the new wave being more recent has an amplitude that exceeds that of the old one by an exponential factor $e^{-2(t-t_0)}$. This effect grows in time, and for times larger than $Me/2$, a new wave field is generated and the trajectory diverges again.

The first and main result is that the availability of intrinsic recorded information about the past can make a temporal reversibility possible for an elementary system even in dissipative and chaotic conditions. Our second result is the finding of a wave erasing process. It gives strength to the description of walker dynamics as an implementation of an iterative computing process. The internal clock is here provided by the periodicity $T_F$ of the Faraday waves and the vertical bouncing motion. At each drop bounce, the generation of a standing Bessel wave can be interpreted as a writing process by which the drop encodes positional information in an extended wave field. Because of the parametric forcing of this wave field, the positional information is maintained for a given time, which corresponds to a storing process in a wave field. At each new bounce, the drop as it comes in close contact with the bath receives a horizontal kick proportional to the local slope. It corresponds to a reading process in which the stored information determines the drop’s next jump. To these three basic operations (writing, storing, and reading), we have added the existence of the fourth basic elementary operation: the erasing process, which can be here triggered by an imposed $\pi$ shift on the bouncing phase of the drop. The specificity and the associated richness of this iterative machine rely here on the way the information is written, stored, and processed. Each individual positional information is stored in a global wave field, submitted to the superposition principle of waves. The walker can in that sense be termed as a wave Turing machine. With the present control of the bouncing phase, the wave memory can be written or erased on demand. This dynamical information storage through a global wave memory can thus be controlled. Even though the present system is unpractical, the finding of similar coupling with waves of a different nature could lead to computing possibilities.

The authors thank M. Labousse and Y. Pomeau for useful discussions, S. Neveu for providing us the ferrofluid we used, and D. Charalampous, A. Lantheaume, and L. Rhea for technical assistance. This work was supported by the French government through Grants No. ANR-11-BS04-001-01, No. ANR-10-IDEX-0001-02 PSL*, and No. ANR-10-LABX-24, and by the AXA Research Fund.

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